Elementary units of exposure

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Abstract

Most road safety studies rely on summary measures of exposure. The term summary measure denotes any aggregate indicator of exposure that does not directly identify and count the number of opportunities for accidents to occur. This paper shows how elementary units of exposure can be developed on the basis of known aggregate measures, such as AADT. An elementary unit of exposure refers to any event that generates an opportunity for an accident to occur. Four such events are identified: (1) Encounters, i.e. vehicles passing each other in opposite directions of travel; (2) Simultaneous arrivals at points of intersection between potentially conflicting directions of travel, in particular vehicles entering intersections at the same time or within a very short time interval; (3) Change of travel lane on multi-lane highways; (4) Braking or stopping. These events describe traffic movements prior to a wide range of crash types. The only major group of accidents that is not directly related to particular events is running off the road. The number of events expected to occur for each of the four types identified is estimated by relying on the assumptions: (A) that AADT is known – when estimating the number of events, mean hourly volume (AADT/24) is used; (B) that vehicles or road users arrive at a point of potential conflict according to a Poisson process, and (C) that simultaneous arrivals within a very short time interval (such as 1 second) have the potential for generating a conflict. It is found that the number of encounters and simultaneous arrivals in intersections increases considerably faster than AADT. The number of events that may generate conflicts involving lane changes or braking or stopping increases more slowly than AADT.

Key words: exposure, traffic events, Poisson process, road user learning

1. Introduction

The concept of exposure and its measurement is a topic with a long history in accident research (1-4). Various definitions of the concept have been proposed. The two most common measures of exposure in current road accident research is vehicle kilometres (or miles) of travel and the number of entering vehicles in intersections (sometimes identified as entering from the major or minor approaches). These measures are normally treated as summary measures, in the sense that only their total values or mean values, usually annual average daily traffic (AADT), are used in analysis. As shown by Mensah and Hauer (5), the use of average values for exposure may lead to problems in estimating the correct shape of the relationship between exposure and the number of accidents. AADT, in particular, is an average covering very different conditions that may involve different levels of risk, like daylight and darkness, fine weather and bad weather, slippery roads and dry roads, etc., etc.

Recent research (6-8) shows that the relationship between summary measures of exposure and the number of accidents is non-linear. This finding invalidates the traditional use of accident rates in safety analyses to control for different levels of exposure. In turn, this has spurred an interest in developing measures of exposure for which the level of risk is constant.
at all levels of exposure. This paper argues that conceiving of risk as being independent of exposure is likely to be wrong for any practically useful measure of exposure. An attempt is made to decompose commonly used summary measures of exposure into elementary units of exposure. An elementary unit of exposure will be defined as any clearly defined and countable event that generates an opportunity for an accident to occur. By identifying elementary units of exposure, it is possible to gain more insight into the mechanisms by which traffic causes accidents, i.e. how road users form expectations and adapt behaviour to specific traffic events that represent a potential for accident occurrence.

2. Foundations in probability theory

It may be useful to briefly revisit the foundations of the concepts of exposure and risk in probability theory. The basic concepts of the branch of probability theory that forms the basis of modern accident research were developed by the French mathematician Simeon Denis Poisson more than 150 years ago. Poisson investigated the properties of binomial trials. He studied what happened to the binomial probability distribution when the number of trials, N, became very large, while at the same time the probability of failure, p, became very low. Denote the expected value in N trials by \( \lambda \) \( (\lambda = N \cdot p) \). Poisson found that the probability of \( x \) failures in N trials could be adequately described by the following probability function, which bears his name:

\[
P (X = x) = \frac{\lambda^x e^{-\lambda}}{x!}
\]

In the terminology of accident research:

Expected number of accidents \( (\lambda) = \) Exposure (N) \( \cdot \) Accident rate (p)

Accident rate is traditionally estimated as the number of accidents per unit of exposure:

\[
\text{Accident rate} = \frac{\text{Number of accidents}}{\text{Unit of exposure}}
\]

In terms of probability theory exposure ought to refer to the number of trials; accident rate ought to refer to the probability of failure at each trial. In practice, however, the estimators used for exposure and risk in safety analyses do not form independent and homogeneous trials for which risk remains constant independently of the number of trials. This point of view is elaborated in the next section.

3. The weaknesses of summary measures of exposure

There are two problems in using accident rates, as defined above, in order to control for the effects of differences in exposure on the number of accidents. The first problem arises from the fact that accident rate is not independent of exposure, but tends to decline as exposure increases. This tendency is most clearly evident in driver accident rates, as shown in recent studies (9-12). Thus in the study of Hakamies-Blomqvist et al. (9), accident rates for drivers aged 26-40 years were:

72.4 accidents per million km of driving for drivers whose mean annual driving distance was 1272 km;

14.7 accidents per million km of driving for drivers whose mean annual driving distance was 8497 km;
5.8 accidents per million km of driving for drivers whose mean annual driving distance was 25536 km.

These accident rates cannot be interpreted as estimates of the probability of accidents. The probability of becoming involved in an accident is not even positively related to the accident rates. The mean annual expected number of accidents can be estimated to 0.092 for low-mileage drivers, 0.125 for middle-mileage drivers and 0.148 for high-mileage drivers (estimated by multiplying accident rate by annual mileage). If the assumption is made that accidents occur according to the Poisson probability law, the probability of becoming involved in at least one accident during a year can be estimated to:

0.088 for drivers who drive a mean annual distance of 1272 km;
0.117 for drivers who drive a mean annual distance of 8497 km;
0.138 for drivers who drive a mean annual distance of 25536 km.

In other words: as exposure increases, so does the probability of becoming involved in an accident, but the each additional kilometre driven becomes safer.

The second problem in computing and using accident rates arises in the case of composite exposure, i.e. exposure consisting of two or more traffic movements that both contribute to the risk of accident. Examples include pedestrians crossing the road (both the number of pedestrians and the number of vehicles contribute to the risk) and turning movements conflicting with traffic going straight ahead in intersections. Hauer (13) illustrates the problem in discussing the effects on safety of providing left turn phases at signalized intersections. The number of accidents involving left-turning vehicles depend both on the number of vehicles turning left and on the number of oncoming vehicles going straight through the intersection. Hauer shows by means of an example that if exposure to the risk of a left-turn accident is estimated by using the number of left-turning vehicles to measure exposure, permissive/protected (lagging) phases (i.e. a left turn signal comes on at a time when the opposite traffic stream still has a green signal) have a lower accident rate than protected/permissive (leading) phases (i.e. a left turn signal comes on when the opposite traffic stream still has a red signal but continues into the green phase). If the sum of left-turning and straight-ahead vehicles is used to measure exposure, leading and lagging phases have identical accident rates. If the product of the two traffic movements is used to measure exposure, leading phases have a lower accident rate than lagging phases. The problem is that it is not obvious which of these measures of exposure, if any, that most correctly reflect the opportunity for accidents to occur.

4. Deriving elementary units of exposure

Exposure can be defined as any event that generates an opportunity for an accident to occur. Elementary units of exposure can, to some extent, be derived from summary measures like AADT by defining specific events that represent opportunities for accidents to occur. In this paper, four such elementary events have been defined:

1. Encounters
2. Simultaneous arrivals from conflicting, or potentially conflicting directions of travel
3. Changes of direction of travel close to other vehicles or road users
4. Braking or stopping

The first two of these events can be modelled as chance events, whereas the latter two are the result of decisions taken by drivers. The decision taken to change lane or brake will not be modelled in this paper. The models developed are based on the assumption that, for whatever reason, a decision has been made to change lane or to brake. The question then
is: given this event, what is the probability that it will result in a potential conflict with other road users?

The general answer to this question is that there is a potential for a conflict if two or more road users arrive at the same place from different directions or at different speeds, at the same time or within a very short time interval. In the following analyses a time interval of 1 second has been chosen. This time interval is regarded as short enough that there is real potential for a conflict to occur. In all analyses, arrivals have been modelled as a Poisson process.

### 4.1 Encounters

Encounters are the passing of vehicles travelling in opposite directions. Each encounter represents an opportunity for a head-on crash on an undivided highway. On divided highways, head-on crashes are still in principle possible, but the opportunities are greatly reduced (14). The number of encounters on an undivided road equals:

\[
\text{Number of encounters} = \left( \frac{\text{Number of vehicles in both directions per unit of time}}{2} \right)^2
\]

If AADT is known, the number of encounters expected to occur at any point on the road can be estimated for any period of time by dividing AADT by 2, and further dividing by, for example, 24 to obtain mean hourly volume. The number of encounters is obtained by raising the number of vehicles passing a point in both directions, divided by 2, to a power of 2.

### 4.2 Arrivals from potentially conflicting directions

Accidents in intersections, or other points where traffic enters from different, potentially conflicting directions, depend on the number of simultaneous arrivals. There are three traffic streams entering a three leg intersection and four traffic streams entering a four leg intersection. A conflict can only occur if vehicles or pedestrians from at least two of the potentially conflicting traffic streams arrive at the intersection (or crossing facility) simultaneously or very close in time. How is the number of potential conflicts related to the number of vehicles arriving within a small time interval?

Estimates have been developed by assuming that:

1. Arrivals per unit of time occur by a Poisson process. An hour has been chosen as a suitable unit of time in all analyses.
2. Arrivals from different approaches are independent of each other.
3. Arrivals within the same 1 second have a potential for generating a conflict.
4. The mean hourly number of arrivals is equal in all approaches.

If these assumptions are made, it is possible to derive a closed-form solution to the problem, given in terms of a general formula for the probability of a conflict.

To show the logic of the analysis, a numerical example will be given. It is assumed that hourly volume in a three-leg intersection is 350. Mean hourly volume per approach is then 116.7. The mean number of arrivals per second per approach is 116.7/3600 = 0.0324. This is the mean value, denoted \( \lambda \), of a Poisson distribution.
The probability of zero arrivals per second per approach is then:

\[ \text{Probability of zero arrivals per second per approach} = e^{-\mu} = M = 0.9681 \]

The probability of zero arrivals in all three approaches is \(0.9681^3 = 0.9074\). If there are two or more simultaneous arrivals, a conflict may occur. The probability of two or more arrivals, given the mean number of arrivals per second above, can be estimated to 0.002986. Similarly, if entering volume had been, for example, 1000 vehicles per hour, the probability of a conflict can be estimated to 0.022079. By relying on the assumptions stated above and applying a Taylor-series approximation, the following formulas for estimating the probability of a conflict have been found:

\[ \text{Probability of conflict in three leg intersection} = 2 \cdot M^3 - 3M^2 + 1 \]

\[ \text{Probability of conflict in four leg intersection} = 3 \cdot M^4 - 4M^3 + 1 \]

\(M\) denotes \(e^{-\mu}\). Figure 1 shows the probability of conflict as a function of entering volume per hour. It should be noted that these formulae apply only when all the assumptions made are valid. If, for example, entering volumes are not evenly balanced, more complex expressions are needed. However, sensitivity analyses have been made that show that the formulae are good approximations even if entering volumes are unbalanced (see the discussion section of the paper).

### 4.3 Changes of direction

The opportunities for accidents to occur when changing lanes on multi-lane highways can be modelled exactly like the opportunities for accidents created by simultaneous arrivals at intersections. The logic is as follows. Assume that each lane has an hourly volume of, say, 200 cars. The decision to change lanes is taken at a given point, which is analogous to the point of entering an intersection. At a volume of 200 cars per lane per hour, the expected number of cars arriving at a certain point in a given lane during an interval of 1 second is \(200/3600 = 0.0556\). If arrivals are assumed be random and follow the Poisson distribution, it can be estimated that the probability of at least one car arriving at the same point during a 1 second interval equals 0.0540. Mean lane volume can be estimated by dividing AADT (or hourly volume) by the number of lanes. The relative probability of a conflict is then equal to:

\[ \text{Relative probability of conflict} = \left( \frac{\text{AADT}}{\text{Number of lanes}} \right)^{0.9} \]

Thus, if lane volume increases from 200 to 600 vehicles per hour, the probability of a conflict increases from 0.0540 to 0.1535. This formula will obviously not always be strictly correct, but as stated in the introduction, the objective of this paper is to derive measures of exposure based on easily available summary statistics, like AADT.

### 4.4 Braking or stopping

The decision to brake or stop can be thought of as analogous to the point of entering an intersection or encountering a vehicle travelling in the opposite direction. Thus, to model the opportunities for accidents generated by braking or stopping, we may conceive of braking as starting when a car passes a certain point on the road. To model how braking or stopping
may generate opportunities for accidents, it is necessary to make specific assumptions regarding traffic volume, driver reaction time, road surface friction and whether braking involves just a slowdown or a full stop.

To simplify, it is assumed that conflicts can only arise between vehicles travelling in the same direction in the same traffic lane. This is not strictly correct, but will allow a general formula for estimating the potential number of conflicts to be derived. Braking to a full stop will be assumed. A uniform driver reaction time of 1.5 seconds is assumed. If headway exceeds reaction time, it is in principle always possible to stop in time, as shown in figure 2. Thus, the task is one of modelling the probability that one or more following cars will arrive within a period of 1.5 seconds at the point where the first car started to brake. A model was developed by assuming that cars arrive according to a Poisson process. Thus:

Expected number of cars arriving at a random point within 1.5 second period = hourly lane volume/3600/1.5 = \( \lambda \)

Probability of 0 cars arriving within 1.5 seconds = \( e^{-\lambda} = M \)

Probability of 1 or more cars arriving within 1.5 seconds = \( M^0 \cdot (1 - M^1) \)

Probability of 2 or more cars arriving within 1.5 seconds = \( M^1 \cdot (1 - \text{Poisson probability of 1 or more arrivals}) \)

The probability of more cars arriving was estimated up to 8 cars, but declined rapidly to zero. Estimating it for more than 8 cars was therefore not regarded as necessary. The following formula for the probability of a rear-end conflict was derived:

Probability of conflict = \( 1 - \lambda \cdot (M^2 + M^3 + M^4) - \frac{\lambda^2}{2} \cdot (M^3 + M^4) + \frac{\lambda^3}{6} \cdot M^4 - M^4 \)

This formula closely approximates the probability estimated by adding the probabilities of 1, 2, 3, ..., 8 arrivals within a 1.5 second period. Figure 3 shows the probability of conflict involving braking.

5. The shape of the relationship between exposure and accidents

For a long time, the standard assumption in road safety research was that the number of accidents, all else equal, increased in direct proportion to traffic volume, generally measured as AADT. This assumption is inherent in any use of accident rates (accidents per million units of exposure) to describe the level of safety. Hauer (4) questioned the widespread and uncritical use of accident rates almost 15 years ago. Subsequent developments in accident modelling have fully borne out his points of view. Nearly all recently developed accident prediction models have found that the relationship between traffic volume and the number of accidents is non-linear, and that its exact shape depends on accident severity and on the type of accidents studied.

However, the shape of the relationship between exposure and accidents is actually more complex than most modern accident prediction models suggest. Most of these models still rely mainly on summary measures of exposure, like AADT. If exposure is conceived of as traffic events that generate an opportunity for an accident to occur, different measures of
exposure are needed that show the number of opportunities generated by specific traffic events.

Consider encounters. A model fitted to Norwegian data (15) suggests that head-on crashes increase in proportion to AADT raised to a power of 1.1 (AADT\(^{1.1}\)). This means that the risk of a head-on crash increases as AADT increases. If, however, the number of encounters is used as the denominator in estimating risk, it is found that risk decreases as the number of encounters increases. Thus, if the risk of a head-on crash is set equal to 1.0 when AADT is 500 (corresponding to 62,500 encounters per day), it declines to 0.536 when AADT is 1,000 (corresponding to 250,000 encounters per day. At an AADT of 20,000, the risk of a head-on crash per encounter is 0.036, if set equal to 1.000 at an AADT of 500. Thus, the more frequently encounters occur, the less becomes the risk of a crash per encounter.

The potential number of conflicts at intersections also increases more rapidly than entering volume. This suggests that the rate of accidents is likely to decline as a function of the volume of exposure, if the potential number of conflicts is used as indicator of exposure to risk. As far as lane changes and braking are concerned, it was found that the probability of a conflict, given that someone changes lane or brakes, increases more slowly than hourly lane volume. It should be born in mind, however, that this analysis did not consider the probability that someone will decide to change lane or brake. It seems likely that the frequency of these events is positively related to hourly traffic volume. Changing lane in rush hour traffic is tempting in order to save travel time. Dense rush hour traffic is also likely to be characterised by stop and go conditions, leading to frequent braking. It is therefore not unreasonable to hypothesise that the potential number of conflicts involving lane changes or braking will increase more rapidly than traffic volume.

6. Discussion

Traffic consists of a set of different, countable events. The events that have been covered in this paper are encounters (vehicles passing each other in opposite directions of travel), simultaneous arrivals from potentially conflicting traffic directions, changes of direction, more specifically lane changes, and braking and stopping. These events represent the number of opportunities for a wide range of accidents: head-on crashes, various types of accidents in intersections, pedestrian crashes (which can be thought of as a kind of intersection accident), sideswipe accidents involving lane changes and rear-end collisions.

There is, however, one important type of accident that cannot readily be associated with any of the elementary events discussed in this paper: single vehicle accidents in which the vehicle runs off the road. The risk of running off the road is perhaps best conceived of as a continuous risk. It may occur at any moment when the vehicle is moving and does not need to be associated with any specific traffic event. The initiating condition for running off the road might of course be a specific event – like swerving to avoid hitting an animal – but it could just as well be a non-traffic event – like the driver falling asleep. It therefore seems sensible to use AADT as a measure of exposure to the risk of running off the road.

In developing the relationships between the elementary units of exposure and the potential number of conflicts, hourly volume and random arrivals during short time intervals have been used throughout. Effectively, the approach taken in this paper assumes that hourly volume can be approximated as AADT/24. This approximation is clearly very crude. It will, however, not necessarily introduce a large bias into the relationships developed. To see this, consider a numerical example.

Suppose a three leg intersection has an evenly balanced entering volume of 50 vehicles per hour in each approach (i.e. a total of 150) during 8 hours each day, 100 entering vehicles per hour during 12 hours each day, and 200 entering vehicles each hour during 4 hours each day. Total entering volume per approach per day will be 2,400, for a mean hourly volume per approach of 100. Total entering volume for the intersection will be 7,200 vehicles per
day. Relying on the formula derived, it can be estimated that the potential number of conflicts per day using a constant volume of 100 vehicles per hour per approach is 191. The potential number of conflicts per day using entering volumes of 50, 100 and 200 vehicles is 233. Not accounting for variations in hourly volume throughout the day results in an underestimate of the potential number of conflicts. Now consider an identical three leg intersection with evenly balanced entering volumes per approach of 200 (8 hours), 400 (12 hours) and 800 (4 hours) vehicles. For this intersection, the potential number of conflicts per day can be estimated to 2665 using a constant hourly volume of 400 entering vehicles and to 3063 using varying entering volume per hour. The shape of the relationship between the potential number of conflicts and hourly volume is found by taking the ratios of these numbers. This ratio is 2665/191 = 13.95 relying on a constant hourly volume throughout the day and 3063/233 = 13.15 relying on varying hourly volumes throughout the day. The difference between these ratios is minor.

In the interest of simplicity, it is therefore suggested to estimate hourly volume simply by dividing AADT by 24. The inaccuracy resulting from this approach is minor and the main point of this paper is to show how event-based measures of exposure can be derived in a simple way even if only AADT is known.

Another simplification made in developing the closed-form solution for intersections was that entering volume was evenly balanced between approaches. If entering volumes are unbalanced, the probability of a conflict is slightly reduced, but the shape of the relationship between total entering volume and the probability of a conflict is the same as when all approaches have the same number of entering vehicles.

According to the classical Poisson probability model of accident occurrence, accidents occur at a constant rate per unit of exposure – or per “trial” to use the terminology of probability theory. If by a trial we think of the events that have been discussed in this paper, it seems clear that accidents do not occur at a constant rate per trial, or event. On the contrary, it is highly likely that for all the measures of exposure discussed in this paper, the relationship between the amount of exposure and the rate of accident per unit of exposure will be inverse: the greater the exposure (number of events), the lower the risk of an accident per event.

It is not surprising that the relationship between exposure and the rate of accidents per unit of exposure is inverse. The inverse nature of the relationship is likely to reflect a very general tendency, referred to by Elvik (16) as a “law of accident causation”, and labelled by him as “the universal law of learning”. Traffic events provide opportunities for learning. The more events of a certain kind we experience as road users, the better we learn how to identify and control the risk involved in those events. In all exposure to road accident risk, human learning is involved. One would therefore expect the tendency for the rate of accidents to decline as a function of the number of opportunities for learning to apply universally.

It might be objected that the events proposed as indicators of exposure in this paper refer to infrastructure elements, like undivided roads (encounters) or intersections (simultaneous arrivals from conflicting directions) and that these elements do not learn anything. However, an intersection tends to be used by more or less the same road users every day, as most travel by road is local. In any intersection, the proportion of local users is likely to be high. Being repeatedly exposed to similar events and situations at the same place, local road users are likely to learn from these experiences. The more they learn, the more reliable becomes their performance and the lower becomes the risk per event representing an opportunity for an accident.

To illustrate the effect of learning, consider an accident prediction model for intersections developed by Jonsson et al. (6). According to the model, the expected number of accidents in a three leg intersection is estimated to increase by a factor of 18.7 if hourly entering volume (assumed to be equal in all approaches) increases from 200 to 1600. Thus, when
entering volume increases by a factor of 8, the predicted number of accidents increases by a factor of 18.7, showing that accident rate, as conventionally estimated, increases. However, the potential number of conflicts increases by a factor of 51.8 when hourly volume increases from 200 to 1600. Since accidents increase by a factor of only 18.7, this shows that the majority of the potential conflicts are resolved in a way that does not result in an accident. The potential increase is by a factor of 51.8, the actual increase only by a factor of 18.7; an index of “learning efficiency” of \((51.8 – 18.7)/51.8 = 0.639\) can be deduced from this – showing that 63.9 % of the potential additional accidents resulting from potential conflicts are avoided.

7. Conclusions

The following conclusions can be drawn from the research reported in this paper:

1. In most road safety studies, only summary measures of exposure, such as AADT, are used. The relationship between summary estimates of exposure and the number of accidents tends to be non-linear.

2. In this paper, four measures of exposure, defined as traffic events that generate the opportunity for accidents, have been derived from summary measures of exposure, by assuming that traffic arrives at points of intersection by a Poisson process and that arrivals that are simultaneous or very close in time represent an opportunity for an accident to occur. All the event-based measures of exposure can be derived from AADT, meaning that their values can be estimated by relying on data that are usually available.

3. The four event-based measures of exposure derived in this paper are: (a) encounters, (b) simultaneous arrivals from potentially conflicting traffic directions, (c) lane changes, and (d) braking or stopping. These measures of exposure can be directly related to specific types of accidents, except for single-vehicle running-of-the-road accidents. It is suggested that AADT can be used as a measure of the exposure to the risk of running off the road.

4. The event-based measures of exposure tend to increase considerably more rapidly than AADT, suggesting that events generating the potential for a conflict occur more and more frequently as AADT increases.

5. The rate of accidents per event-based unit of exposure is very likely to fall considerably as the number of events increases. This reflects the fact that as road users repeatedly and frequently experience the same or similar events, they learn how to identify and control the risks involved in those situations.

References


Figure 1: Probability of conflict as a function of entering volume in intersections
Figure 2: Logic of accumulation of reaction times in braking situations

- Time elapsed (seconds)
- Distance covered (metres)

First driver comes to a complete stop (4.6 seconds = 65.2 metres)

Second driver comes to a complete stop (6.1 seconds = 65.2 metres)

End of second driver’s reaction time

Earliest possible start of second driver’s reaction

Reaction time of first driver (1.5 seconds = 33.3 metres)
Figure 3: Probability of conflict involving braking as a function of hourly lane volume